Problem 12-9

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)$ m/s^2, where t is in seconds. If $s = 1$ m and $v = 2$ m/s when $t = 0$, determine the particle's velocity and position when $t = 6$ s. Also, determine the total distance the particle travels during this time period.

Solution

The acceleration and velocity are related by

$$a = \frac{dv}{dt} = 2t - 1.$$

Integrate both sides with respect to t to get the velocity.

$$\begin{aligned} v(t) &= \int (2t - 1)\, dt \\ &= t^2 - t + C_1 \end{aligned}$$

Use the fact that $v = 2$ when $t = 0$ to determine C_1.

$$v(0) = C_1 = 2$$

The particle's velocity (in meters per second) is then

$$v(t) = t^2 - t + 2.$$

The velocity and position are related by

$$v = \frac{ds}{dt} = t^2 - t + 2.$$

Integrate both sides with respect to t to get the position.

$$\begin{aligned} s(t) &= \int (t^2 - t + 2)\, dt \\ &= \frac{t^3}{3} - \frac{t^2}{2} + 2t + C_2 \end{aligned}$$

Use the fact that $s = 1$ when $t = 0$ to determine C_2.

$$s(0) = C_2 = 1$$

The particle's position (in meters) is then

$$s(t) = \frac{t^3}{3} - \frac{t^2}{2} + 2t + 1.$$

Therefore,

$$\begin{aligned} s(6) &= \frac{6^3}{3} - \frac{6^2}{2} + 2(6) + 1 = 67 \text{ m} \\ v(6) &= (6)^2 - (6) + 2 = 32 \, \frac{\text{m}}{\text{s}}. \end{aligned}$$

The total distance the particle travels until $t = 6$ s is the integral of the speed from $t = 0$ to $t = 6$.

$$\begin{aligned} s_T &= \int_0^6 |v(t)|\,dt \\ &= \int_0^6 |t^2 - t + 2|\,dt \\ &= \int_0^6 (t^2 - t + 2)\,dt \\ &= \left(\frac{t^3}{3} - \frac{t^2}{2} + 2t\right)\Big|_0^6 \\ &= \frac{6^3}{3} - \frac{6^2}{2} + 2(6) \\ &= 66 \text{ m} \end{aligned}$$